OPTIMIZATION OF THE CONDITIONS OF MACHINING PARAMETERS BASED ON A CRITERION COMBINED BY GENETIC ALGORITHMS

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Abstract

In metal cutting processes, cutting conditions have an influence on reducing the production cost and time and deciding the quality of a final product. This paper deals with the multiple-objective optimization of machining conditions problem in order to optimize the time and production cost simultaneously. This method consists essentially of improving previous methods by introducing the Pareto technique for solving the multiple-objective optimization problem using Genetic Algorithm. This method has been used for single pass turning.

A comparison between results obtained by the proposed approach and those obtained with the gradient and the simplex methods was carried out.

NOMENCLATURE

\(a_1, a_2, a_3\) \hspace{1em} Empirical constants for tool life equation.
\(C_0\) \hspace{1em} Operating cost (\$/min)
\(C_t\) \hspace{1em} Tool cost per cutting edge (\$/edge)
\(C_u\) \hspace{1em} Production cost (\$/piece)
\(d_c\) \hspace{1em} Depth of cut for a pass (mm)
\(f\) \hspace{1em} Feed rate (mm/tour)
\(F\) \hspace{1em} Cutting force (N)
\(HP\) \hspace{1em} Machine horsepower (kW)
\(K\) \hspace{1em} Tool life constant.
\(L\) \hspace{1em} Workpiece length to be machined (mm)
\(SR_{\text{max}}\) \hspace{1em} Maximal surface roughness (µm)
\(t_{cs}\) \hspace{1em} Tool change time (min./edge)
\(t_h\) \hspace{1em} Loading and unloading time (min)
\( t_m \quad \text{Machining time (min)} \)
\( t_g \quad \text{Quick return time for the last pass (min / pass)} \)
\( T \quad \text{Tool life (min/edge)} \)
\( V \quad \text{Cutting speed (m/min)} \)
\( U \quad \text{Objective function.} \)
\( W_1, W_2 \quad \text{Weight coefficients.} \)
\( \theta \quad \text{Average cutting temperature at the tool-chip interface,(deg C)} \)

**Introduction**

The optimization of the cutting conditions consists in search of the optimal values of the machining parameters (cutting speed, feed, etc.), with respect to a certain criterion (the production cost, the time of production, the productivity, etc.). The interest of the optimization of the cutting conditions lies in the reduction of the cost of the production, the increase in the productivity, the improvement of the dimensional accuracy of the parts and the simplification of the programs of machining on the machine tools with numerical control.

Among rare work of optimization of the conditions using the multiple-objective optimization the method proposed by J.S.Agapiou [1] is of particular interest.

Our task is to develop the ideas presented by the cited approach and to contribute our share by looking for the best solutions for multiple-objective optimization using genetic algorithms (G.A). The genetic algorithms are algorithms of exploration based on the mechanisms of the natural selection and the genetics. They effectively exploit information obtained previously to speculate in the position of new points to explore, with the hope to improve the performance.

**Basic genetic algorithm operations:**

Genetic Algorithm is based on simple string copying and substring concatenation [5], [6], [8]. There are three basic operators found in every genetic algorithm: reproduction, crossover and mutation.

**Stages of G.A**

The stages of AG are as follows: (figure 1)

1. Creation of the initial population.
2. Evaluation of each chromosome of the initial population.
3. Selection and regrouping of the chromosomes per pairs.
4. Application of the crossover and the mutation operators.
5. New evaluation of the chromosomes and insertion in the following population.
6. If the criterion of stop is reached, the genetic algorithm stops and returns the best chromosome produced; if not, the algorithm turns back at the stage 3.

**Reproduction:**

This operator known as “strategy of the elitism ” consists in recopying the best chromosome of the current population in the population of the following generation, and to supplement it by other generated chromosomes in a traditional way until obtaining the necessary number of individuals.

![General principle of genetic algorithm](image_url)

**Crossover:**

This operator combines the chromosomes of two individuals to obtain two new.

A crossover in a point is obtained in two stages:

- Random choice of an identical point of cut on the two chromosomes.
• Cut of the two chromosomes (figure 2) and exchanges of two fragments located on the right.

The crossover “1 point” is simple and most traditional for binary coding. The crossover “1 point” and “2 points” (figure 3) are usually employed in practice for their simplicity and their good effectiveness.

\[\begin{array}{cccccccccccc}
1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\
\end{array}\]

\[\begin{array}{cccccccccccc}
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0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\
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0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\
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0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\
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\[\begin{array}{cccccccccccc}
1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
\end{array}\]

Figure 2: Representation of crossover in 1 point

Figure 3: Representation of crossover in 2 points

**Operator of mutation:**

We define a mutation as being the inversion of a bit in chromosomes (fig 4). The mutation ensures a random local research around each individual. In this order of idea, the mutation can improve considerably quality of the discovered solutions.

\[\begin{array}{cccccccccccc}
1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\
\end{array}\]

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\end{array}\]

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\end{array}\]

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\end{array}\]

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\end{array}\]

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0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\
\end{array}\]

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1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\
\end{array}\]

Figure 4: Representation of mutation
Formulation of the model of optimization:

The Formulation of the model of optimization requires the knowledge of some mathematical equations representing the economic and physical parameters process of machining represented by the system Part-Machine-Tool [1], [2], [4], [9].

Among the principal criteria of optimization one can use the production cost and the time of production. The analysis of the relation between these two criteria and the cutting conditions allows as to make the following conclusions:

- The more reduced cutting conditions are, the more necessary execution times of the various operations are raised and this leads to the increase in the production cost.
- The increase of cutting conditions causes the fast wear of the tools. Therefore their changes lead to the increase in the production cost.

In general previous studies also showed that the satisfaction of two or several criteria at the same time is practically impossible, since certain criteria are sometimes even competitor (the minimization of the time of production by the use of the cutting conditions causes the increase in the production cost because of the wear of tools. Also the increase in the productivity leads to non respect of the precision of machining, etc...).

Optimization with multiple objectives is currently a very active field of research. The method used consists in defining an objective function by combining the time criterion and production cost in the same function. A compromise thus is established. The time of production and the production cost are combined to formulate the objective function.

The objective function can express by a weighted sum of the objective functions:

\[ f = \sum \alpha_i f_i \]  \hspace{1cm} (1)

Where \( \alpha_i \) represents the weight given associated to \( f_i \). It shows the relative importance of each criterion. It is thus to the user to suitably choose the weights \( \alpha_i \).

One can often classify the objectives according to their importance but the weights will be generally found by trial and error [3].
But this solution is only not very satisfactory. For better answering the problem, we use another technique that the combination of the functions which is the technique of the optima of Pareto [6].

**Time of production:**

Total time required to produce a part is the sum of times necessary for machining, tool changing, tool quick return, and workpiece handling.

\[ T_U = t_m + t_{cs} \left( \frac{t_m}{T} \right) + t_R + t_h \]  

(3)

where \( t_m \) machining time

\( t_{cs} \) tool change time

\( T \) tool life

\( t_R \) quick return time for the last pass.

\( t_h \) loading and unloading time.

\[ t_m = \frac{\pi D L}{1000 V f} \]  

(4)

The tool life \( T \) is given by the following Taylor’s equation:

\[ V f^{a1} d_c^{a2} T^{a3} = K \]

From where one can draw:

\[ T = K^{\frac{1}{a3}} V^{\frac{1}{a5}} f^{\frac{a1}{a5}} c^{\frac{a2}{a3}} \]  

(5)

Subsisting \( t_m \) and \( T \) by their respective expressions given by (4) and (5) into equation (3) we obtain the time of production in the following form:
\[ T_U = A V^{-4} f^{-1} + A V^{(1-a_1)/a_3} f^{(a_1-a_3)/a_3} A f^{(a_2-a_3)/a_3} K c^{-(1/a_3)} t_\alpha + t_h + t_R \quad (6) \]

where \[ A = \frac{\pi D L}{1000} \]

Figure (5) represents the graphical representation of the production time according to feed rate \( f \) and cutting speed \( V \) for a depth of cutting given equal to 2.54 mm.

\[ \text{Figure 5 : Graphical representation of production time.} \]

The production cost is given by the following formula:

\[ C_U = C_0 t_m + \left( \frac{t_m}{T} \right) \left( C_0 t_c + C_1 + C_0 t_h + t_R \right) \quad (7) \]

Following the same procedure as for time of production, we can obtain the expression for \( C_U \) when one replaces \( t_m \) and \( T \) by their expressions given by equation (4) and (5) into relation (7):

\[ C_i = c_0 A V^{a_1} f^{1-a_1} + A V^{(1-a_3)/a_3} f^{(a_1-a_3)/a_3} A f^{(a_2-a_3)/a_3} K c^{-(1/a_3)} (C_0 t_\alpha + C_i) + C_0 (t_h + t_R) \quad (8) \]
Figure (6) represents the function of the production cost according to feed rate \( f \) and cutting speed \( V \) for a depth of cutting given equal to 2.54 mm.

![Graphical representation of cost time.](image)

**Figure 6**: Graphical representation of cost time.

**Constraints:**

Concerning the constraints, we have taken those used by J.S.Agapiou \[1\], \[2\] in order to compare his results with ours.

a) Limitations on feed rate.
   \[
   f \geq f_{\text{min}} \quad \text{and} \quad f \leq f_{\text{max}}
   \]

b) Limitations on the cutting speed:
   \[
   V \geq V_{\text{min}} \quad \text{and} \quad V \leq V_{\text{max}}
   \]

c) Limitations on the depth of cutting:
   \[
   d \geq d_{\text{min}} \quad \text{and} \quad d \leq d_{\text{max}}
   \]

d) Limitations on the allowed maximum power by the machine tool:
   \[
   0.0373f^{-0.91}V^{0.78}d^{0.75} \leq HP_{\text{max}}
   \]

e) Limitations on the surface quality
   \[
   14.785V^{-1.52}f^{-1.004}d^{0.25} \leq SR_{\text{max}}
   \]

f) Limitations on the temperature of cutting:
   \[
   74.96V^{0.4}f^{-0.2}d^{0.105} \leq \theta_{\text{max}}
   \]

g) Limitations on the cutting force:

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The constants and the coefficients of the mathematical models of the constraints above and the tool life are obtained in experiments by the method of multi-factorial analysis.

To impose the respect of the constraints of a problem, several approaches acting on the various genetic operations of the algorithm are usable. The solution that we used consists in calculating the function of adaptation only in the realizable space of the phenotypes. The individuals of unrealizable space see themselves affecting a bad adaptation which will prevent their reproduction. It is about the method of the death penalty (death penalty method).

**Example of resolution:**

The mathematical model of optimization is composed of the objective function and some of constraints or limitations. The parameters used for the numerical application are mentioned in table 1. For the resolution of this problem by the method of the genetic algorithms, we used a program in language Fortran 90 [7], after to have added some modifications.

The application of this program makes it possible to find best results (parameters which correspond to a minimum time and cost) afterwards the flow of 100 generations and with the use of the strategy of division and the strategy of elitism with a uniform crossover. The problem is represented by two variables the cutting speed and the feed rate which are represented by two genes, each gene having a length of 15 bits, on the other hand for the values of the rate of crossover and mutation, we took for the crossover in only one point a rate equal to 0.7 and one uniform crossover equalizes to 0.5. For the mutation the rates are calculated by the following formula: \( Rate \text{ of mutation} = \frac{1}{\text{population number}} \)

The results found by using the genetic algorithm are mentioned on table 2. Tables 3 and 4 gives the results found by the methods used by Agapiou [1] and simplex method used by Assas. M. and Djenane [9] respectively.

\[ 844V^{-0.1013} \cdot f^{0.725} d^{0.75} \leq F_{\text{max}} \]
Tableau 1 : cutting parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$ (mm)</td>
<td>203</td>
<td>$t_R$ (min/pass)</td>
<td>0.13</td>
</tr>
<tr>
<td>$D$ (mm)</td>
<td>152</td>
<td>$t_h$ (min/part)</td>
<td>1.5</td>
</tr>
<tr>
<td>$V_{\text{min}}$ (m/mn)</td>
<td>30</td>
<td>$\theta_{\text{max}}$ ($^\circ$C)</td>
<td>500</td>
</tr>
<tr>
<td>$V_{\text{max}}$ (m/mn)</td>
<td>200</td>
<td>$a_1$</td>
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</tr>
<tr>
<td>$f_{\text{min}}$ (mm/rev)</td>
<td>0.254</td>
<td>$a_2$</td>
<td>0.35</td>
</tr>
<tr>
<td>$f_{\text{max}}$ (mm/rev)</td>
<td>0.762</td>
<td>$a_3$</td>
<td>0.25</td>
</tr>
<tr>
<td>$SF_{\text{max}}$ ($\mu$m)</td>
<td>2</td>
<td>$K$</td>
<td>193.3</td>
</tr>
<tr>
<td>$SR_{\text{max}}$ ($\mu$m)</td>
<td>2</td>
<td>$t_{cs}$</td>
<td>0.5</td>
</tr>
<tr>
<td>$HP_{\text{max}}$ (kW)</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>$C_0$ ($$/$$/min)</td>
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<tr>
<td>$C_f$ ($$/$$/edge)</td>
<td>0.5</td>
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Tableau 2. Computation results of optimal cutting parameters using genetic algorithm

<table>
<thead>
<tr>
<th>$d_c$ (mm)</th>
<th>$f$ (mm/rev)</th>
<th>$V$ (m/mn)</th>
<th>$C_u$ ($)</th>
<th>$T_u$ (min)</th>
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<tr>
<td>1.27</td>
<td>0.762</td>
<td>132.55</td>
<td>0.377</td>
<td>2.69</td>
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<td>0.634</td>
<td>122</td>
<td>0.525</td>
<td>3.09</td>
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<tr>
<td>3.81</td>
<td>0.507</td>
<td>115.08</td>
<td>0.667</td>
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<tr>
<td>5.08</td>
<td>0.444</td>
<td>107.84</td>
<td>0.773</td>
<td>4.02</td>
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</table>

Tableau 3. Computation results of optimal cutting parameters using simplex method [9]

<table>
<thead>
<tr>
<th>$d_c$ (mm)</th>
<th>$f$ (mm/rev)</th>
<th>$V$ (m/mn)</th>
<th>$C_u$ ($)</th>
<th>$T_u$ (min)</th>
</tr>
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<tbody>
<tr>
<td>1.27</td>
<td>0.76</td>
<td>135</td>
<td>0.382</td>
<td>2.68</td>
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<td>2.54</td>
<td>0.636</td>
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<td>0.525</td>
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<td>3.81</td>
<td>0.554</td>
<td>118.78</td>
<td>0.688</td>
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<td>5.08</td>
<td>0.461</td>
<td>110.50</td>
<td>0.795</td>
<td>3.934</td>
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Tableau 4. Computation results of optimal cutting Parameters using the gradient method [1]

<table>
<thead>
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<th>$d_c$ (mm)</th>
<th>$f$ (mm/rev)</th>
<th>$V$ (m/mn)</th>
<th>$C_u$ ($)</th>
<th>$T_u$ (min)</th>
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<td>173</td>
<td>177</td>
<td>0.504</td>
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<td>2.54</td>
<td>0.56</td>
<td>145</td>
<td>149</td>
<td>0.675</td>
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<td>137</td>
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<td>5.08</td>
<td>0.38</td>
<td>128</td>
<td>131</td>
<td>1.032</td>
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</table>

Conclusion:

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While studying several methods of optimization of machining parameters, our interest was related particularly to the model presented by J.S. Agapiou [1] which represents one of the rare approaches for the resolution of multi-objective optimization. We made a clear improvement of this method by using the method of the genetic algorithms for the research of the optimums conditions for single pass turning. We used the technique of optimum of Pareto for the resolution of such kind of optimization problem.

Our computation results for the same case of application show a clear reduction in the costs and time of machining compared to the simplex method [9] and the gradient method used by Agapiou [1].

References: