A 2D Model of Induction Machine Dedicated to faults Detection– Extension of the Modified Winding Function

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Abstract – This paper treats mainly the modeling of induction machine inductances taking into account all the space harmonics, and with introduction of the effects of skewing rotor bars and linear rise of MMF across the slot. The model is established initially in the case of symmetric machine, which corresponds to the case of a constant air-gap, then in the other case where the machine can present a static or dynamic, axial or radial eccentricity. This objective would be achieved by exploiting an extension in 2-D of the modified winding function approach (MWFA). It should be noted that this work gives a great number of results with consequently less explanation. It is of course our first aim; this can be noticed by a reader already leaning on the subject.

Index Terms – Induction machines, inductance, MWFA, spaces harmonics, skew.

I. INTRODUCTION

The multiple coupled circuit, defined in the aim of approaching the real structure of the rotor cage, supposes that this one gathers a number of loops forming a polyphase winding, each loop consists of two adjacent bars and the two portions of the end ring which connect them [1]. Such a structure was used with profit in the diagnosis of the induction machine. Several studies were carried out in this axis, and made possible to reveal some phenomena rising from a defect, such as appearance of higher or lower sideband frequencies than the stator frequency in the spectral analysis of the line currents, torque, speed and power. Some papers supposes a perfect distribution of the MMF in the air-gap, others adopt models taking into account the real distribution of machine’s windings [2], in particular with the implication of winding function, then, MWFA [3], where it is possible to detect some phenomena accompanying a probable eccentricity. And finally, the introduction of the axial dimension [4],[5]. The model is thus ready to define inductances of a machine taking into account the skew of the slots, and which can be extended to the study of other types of axial asymmetries, namely, the axial eccentricities.

In this work, a 2-D model of the induction machine will be approached while focusing the study on its first aim; the modeling of induction machine with non-sinusoidal distribution of the stator winding, and non-uniformity of the air gap. Simulation results as well as comments will be exposed.

II. A 2-D PRESENTATION OF THE MODIFIED WINDING FUNCTION APPROACH

To formulate the problem, we refer to Fig. 1 which gathers two cylindrical masses separated by an air-gap, one of it hollow and represent the stator, and the other represents the rotor. Given abcd an arbitrary contour definite in comparison to a reference of phase fixed on the stator, to an axial reference and to the mechanical position of the rotor measured by respecting a fixed stator reference. For a position θ₀ of the rotor, and at φ₀ = 0 and z₀ = 0 are the points a and b, and at φ and z are c and d. On another side, a and d are located on the stator inner surface, and b and c on the rotor external surface.

Let us take again the same stages of [3] but with the utilization of the axial dimension. Thus, according to the Gauss’s law, the integral of the magnetic flux density on closed surface S of a cylindrical volume defined in comparison to the average radius of the air-gap r is null:

$$ \oint_{S} \mathbf{B} \, ds = 0. $$

By defining, at any of coordinates (φ, z), the magnetic field intensity \( \mathbf{H} \), the magnetomotive force \( F \) and the effective air-gap function \( g \), such as \( \mathbf{B} = \mu_{0} \mathbf{H} \) and \( \mathbf{H} = F / g \),
in this case (1) become
\[ \mu_0 \int_{\Omega} \int_{0}^{2\pi} F(\varphi, z, \theta) \, dz \, d\varphi = 0, \]
(2)
where \( l \) is the effective length of the air-gap. On another side, and according to the Amper's law, it is possible to write
\[ \oint_{a b c d a} H(\varphi, z, \theta) \, dl = \int_{\Omega} JdS. \]
(3)
\( \Omega \) is a surface enclosed by the closed path \( a b c d a \), and \( J \) the current density. According to the MMF and the number of turns enclosed by the closed path \( a b c d a \) and traversed by the same current \( i \), (3) can be written as
\[ F_{ab}(0,0,\theta_r) + F_{bc} + F_{cd}(\varphi, z, \theta_r) + F_{da} = n(\varphi, z, \theta_r)i, \]
(4)
where \( n(\varphi, z, \theta_r) \) is called the 2-D spatial winding function. Following way:
By introducing the average value of the inverse air-gap function \( g^{-1}(\varphi, z, \theta_r) \) with
\[ \langle g^{-1}(\varphi, z, \theta_r) \rangle = \frac{1}{2\pi} \int_{0}^{2\pi} \int_{0}^{l} g^{-1}(\varphi, z, \theta_r) \, dz \, d\varphi, \]
and while exploiting (2) and (5), it will be possible to lead to the expression giving \( F_{cd}(\varphi, z, \theta_r) \) such as
\[ F_{cd}(\varphi, z, \theta_r) = n(\varphi, z, \theta_r)i - \frac{1}{2\pi} l(g^{-1}(\varphi, z, \theta_r)) \int_{0}^{2\pi} \int_{0}^{l} n(\varphi, z, \theta_r)g^{-1}(\varphi, z, \theta_r) \, idz \, d\varphi. \]
(7)
The 2-D winding function can be obtained by dividing the members of (7) by \( i \):\n\[ N(\varphi, z, \theta_r) = n(\varphi, z, \theta_r) - \frac{1}{2\pi} l(g^{-1}(\varphi, z, \theta_r)) \int_{0}^{2\pi} \int_{0}^{l} n(\varphi, z, \theta_r)g^{-1}(\varphi, z, \theta_r) \, dz \, d\varphi. \]
(8)
It is to be noticed that this new expression does not hold any restriction as for the axial uniformity, in particular in term of skewing slots and axial air-gap non uniformity.

III. CALCULATION OF INDUCTANCES

A. Machine with uniform air-gap

Firstly, supposing that the machine is symmetrical. The air-gap length \( g \) is reduced to \( g_0 \), which is the radial air-gap length in the case of no eccentricity. If \( F \) is the MMF distribution in the air-gap due to the current \( i_{A_i} \) flowing in an arbitrary coil \( A_i \), and knowing that the elementary flux corresponding in the air-gap is measured in comparison to an elementary volume of section \( ds \) and length \( g_0 \) such as
\[ d\phi = \mu_0 Fg_{0}^{-1}ds, \]
the calculation of total flux returns to a calculation of double integral. By carrying out the change of variable \( x = r \varphi \) and \( x_r = r \theta_r \), it is as if the study referred to a reference with axes \( X \) and \( Z \) where it is possible to imagine a plane representation of the machine. It is clear that, in this case, \( x \) translate correctly the linear displacement along the arc corresponding to the angular opening \( \varphi \). In the same way concerning \( x_r \) and \( \theta_r \).

Knowing that \( N \) is the MMF per unit of current, the expression giving the flux seen by all the turns of coil \( B_j \) of winding \( B \) due to \( i_{A_i} \) flowing in coil \( A_i \) will be reduced as
\[ \phi_{B_j A_i} = \frac{\mu_0}{g_0} \int_{x_{i_j}}^{x_{i_j+1}} \int_{z_{j_i}}^{z_{j_i+1}} N_{A_i}(x, z, x_r) n_{B_j}(x, z, x_r) i_{A_i} \, dx \, dz. \]
(10)

That is due to the fact that by taking account the axial asymmetry, \( n_{B_j}(x, z, x_r) \) will be defined so as to be able to translate the skew of the slots. In 2-D, it will be written in the following way:
\[ n_{B_j}(x, z, x_r) = \begin{cases} w_{B_j} \langle x(x_{j+1}, z_{j+1}) \rangle, & \text{in the remaining interval} \end{cases}, \]
(11)
where \( w_{B_j} \) is the number of turns of coil \( B_j \). It is equal to 1 in the case of a rotor loop. Generally, the total flux \( \psi_{BA} \) relating to all coils composing winding \( A \) and \( B \) holds its general expression by integrating over the whole surface. And knowing that the mutual inductance \( L_{BA} \) is the flux \( \psi_{BA} \) per unit of current, yields
\[ L_{BA}(x_r) = \frac{\mu_0}{g_0} \int_{0}^{l} \int_{0}^{l} N_{A_i}(x, z, x_r) n_{B_j}(x, z, x_r) \, dx \, dz. \]
(12)
Let us notice that a rearrangement of (12) makes possible to define an inductance defined per unit of length as described in [4]:
\[ L_{BA}(x_r) = \int_{0}^{l} L_{BA}'(z, x_r) \, dz. \]
(13)
where the amount of static and dynamic eccentricity respectively which are function of \( z \). A numerical calculation makes possible to find the integral (15), however, an analytical resolution must call upon an approximated expression of \( g^{-1} \) by carrying out a development in Fourier series. A perfect result would be obtained while stopping at the third term, such as:

\[
g^{-1}(x, z, x_r) \approx P_0(z) + P_1(z) \cos(x/r - \rho) + P_2(z) \cos(2(x/r - \rho))
\]

\( \rho \) and coefficients \( P_0, P_1 \) and \( P_2 \) are calculated from \( g_0, \delta_0, \delta_1(z) \) and \( \theta \) as describing in [7] and [8]. It is to be recalled that for any winding \( A \) and \( B \), equality \( L_{AB} = L_{BA} \) is always checked [7], and that all was calculated compared to an average radius of the air-gap \( r \) in the case of no eccentricity, while admitting that the variations in the radius of the air-gap \( R \) due to the eccentricity are negligible in front of the radius itself, which is not the case concerning \( g \), it is what can be translated as

\[
R(x, z, x_r) = r + \Delta R(x, z, x_r) \approx \frac{r}{g_0 \pm \Delta g(x, z, x_r)}
\]

IV. SIMULATION RESULTS

A. Machine with uniform air-gap.

The first induction machine studied in this paper is a three-phase, 4-pole motor [4], whose parameters appear in the Appendix, and the structure of the stator coils of Fig. 4, where only phase A is represented, and one of its coils numbered. Each circle represents the section of an elementary coil of \( w \) turns. A, B, and C are the three stator phases, and \( r_j \) the \( j^{th} \) rotor loop.

Fig. 5 illustrates the functions which describe the mutual inductances \( L_{r_1A} \) between the first stator phase A, and the first rotor loop, in the four cases considered, without and with taking into account the slots opening and the skewing of rotor bares. A rotor loop is regarded as being a coil with one turn. Note that the mutual inductance between phase and the second rotor loop is the same as giving in Fig. 5, but shifted to the left by \( 2\pi/Nb \). As for the other inductances, \( L_{r_1B} \) and \( L_{r_1C} \) are identically reproduced, but shifted to the right by \( \pi/3 \). The mechanical angle of
skewing of the rotor bars is $\gamma = \pi / 12$ rad, which is selected equal to one stator slot pitch [6], and the width of slot opening $\beta = \pi / 24$. In each figure, the function whose maximum value is most significant, represents the first derivative of the curve of mutual inductance $L_{r1A}$.

As for the self and the mutual inductances between windings of the same frame (stator or rotor), it should be noted that those are not affected by the effect of skewing, however, a variation in the values of the stator inductances is noted, and it is due to the taking into account of the linear rise of MMF across the slot (Table I).

**TABLE I**

<table>
<thead>
<tr>
<th>Beta</th>
<th>$L_A$ (H)</th>
<th>$L_{AB}$ (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 0$</td>
<td>0.1198</td>
<td>-0.0532</td>
</tr>
<tr>
<td>$\beta = \pi / 24$</td>
<td>0.1165</td>
<td>-0.0529</td>
</tr>
</tbody>
</table>

**B. Machine with an eccentric rotor**

1) *Radial eccentricity:* The second specific induction motor studied is a three-phase, 11kw, 50Hz, 4-pole motor, having four coils per phase group, eight coils per phase, series connected [10]. The others parameters are given in the Appendix. Figs. 6, 7, 8, 9, 10 and 11show the results of simulation for different degrees of eccentricity, and with three terms $P_0, P_1$ and $P_2$ used in the development of $g^{-1}$.

![Fig. 6. $L_{r1A}$ and $dL_{r1A}/d\theta_r$ for: a) symetric, b) $\delta_s = 35\%$, $\delta_d = 0$](image)

![Fig. 7. Mutual inductances for: a) $\delta_s = 35\%$, $\delta_d = 0$](image)

![Fig. 8. Case of mixed eccentricity of $L_{r1A}$, $L_{r1B}$, $L_{r1C}$ for: a) $\delta_s = 35\%$, $\delta_d = 0$ b) $\delta_s = 0\%$, $\delta_d = 50\%$](image)
Fig. 9. Self inductance of phase A for different degrees of dynamic eccentricity.

Fig. 10. Self inductance of rotor loop $r_1$ for 50% of static eccentricity.

Fig. 11. Mutual inductance $L_{r1A}$ for different degrees of eccentricity,
  a) Static eccentricity, b) Dynamic eccentricity.

2) Axial eccentricity: To examine the case of the static eccentricity, the expression of $\delta_s(z)$ must be defined. According to Fig. 12 showing the external diameter of the rotor and the internal diameter of the stator with exaggeration in the representation of the air-gap, $\delta_s(z)$ can be written as

$$
\delta_s(z) = \delta_{s0}(1 - \frac{z}{L}).
$$

As presented in Fig. 12, the minimum air-gap for $z = 0$ is supposed at $\varphi = 0$ along the vertical axis. The minimal air-gap has a fixed angular position for the different values of $z$ inferior than $L$, but its value depends on $z$.

On another side, if the perfectly concentric section of the rotor corresponds to $z = L$, in this case, in the modeling of the eccentricity, $L$ must be selected superior than a certain value guaranteeing the existence of an air-gap with $g(x, z, x_i) \neq 0$ along the rotor length. For $L \to +\infty$, as a result $\delta_s(z) \to \delta_{s0}$, and the study returns to the case of the purely radial eccentricity. Fig. 13 shows the mutual inductance between stator phase A and the first rotor loop with rotor position when $\delta_{s0} = 70\%$ and $L = \frac{1}{2}$.

C. Operation under condition of mixed eccentricity.

Knowing that the cage can be viewed as identical and equally spaced rotor loops, it is possible to establish voltage equations of stator and rotor loops as [1],[2]:

$$
\left[ U_s \right] = \left[ R_s \right] \left[ I_s \right] + \frac{d\left[ \psi_s \right]}{dt},
$$

$$
\left[ 0 \right] = \left[ R_r \right] \left[ I_r \right] + \frac{d\left[ \psi_r \right]}{dt},
$$

$$
\left[ \psi_s \right] = \left[ L_{rs} \right] \left[ I_s \right] + \left[ L_{sr} \right] \left[ I_r \right],
$$

$$
\left[ \psi_r \right] = \left[ L_{rs} \right] \left[ I_s \right] + \left[ L_{rr} \right] \left[ I_r \right].
$$

The vector $\left[ U_s \right]$ corresponds to the stator voltages, $\left[ I_s \right]$ and $\left[ I_r \right]$ to the stator and rotor currents. $m$ is the number of stator phases and $N_b$, the number of rotor bars. $\left[ R_s \right]$ is an $m \times m$ dimensional diagonal matrix, $\left[ L_{rs} \right]$ is an $m \times m$ symmetric
matrix, $[L_r]$ is an $m \times (N_b + 1)$ matrix, and $[R_r]$ and $[L_m]$ are $(N_s + 1) \times (N_b + 1)$ matrix. Adding to these equations the mechanical and electromagnetic torque equations:

$$C_e - C_r = J_r \frac{d\omega_r}{dt}, \quad C_e = \left( \frac{dW_{co}}{d\theta_s} \right)_{(I_s, I_r = \text{constant})}, \quad (24)$$

and

$$W_{co} = \frac{1}{2} \left( [I_s] \, [L_{ss}] \, [I_s] + [I_s] \, [L_{sr}] \, [I_r] + [I_r] \, [L_{rr}] \, [I_r] \right) + [I_s] \, [L_m] \, [I_s], \quad (25)$$

where $W_{co}$ is the coenergy, $C_e$ the electromagnetic torque, $C_r$ the load torque, $J_r$ the rotor load inertia, and $\omega_r$ the mechanical speed of the rotor. Fig. 14 shows the simulation result of the operation of machine (2) under conditions of mixed eccentricity of $\delta_s = 40\%$, $\delta_d = 20\%$.

$$\textit{REFERENCES}$$


